Section 6.8 Formulas, applications and variations.

Example 1 pg 409

$$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} \qquad \text{Solve for } r_1$$

Multiply by the LCD, Rr_1r_2 thus: $\left(\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}\right)Rr_1r_2$ to get

$r_1r_2 = Rr_2 + R_1$ Get all the r_1 on one side

$r_1r_2 - Rr_1 = Rr_2$ Factor

$r_1(r_2 - R) = Rr_2$

$$r_1 = \frac{Rr_2}{(r_2 - R)}$$

Notice that we have ONLY one r_1 in the entire equation. This is because we were solving the formula for that value.

Remember: We *NEVER* cross multiply! We always show the LCD we are using to clear fractions.

Variation

Variation problems are in two forms.

Direct

"y" varies directly as "x" translates to: $y = kx$ where k is called the constant of variation.

The concept can be illustrated as …

If we buy five candy bars at 50¢ each we spend $2.50. If we buy eight candy bars, we spend $4.00. The more candy bars we buy, the higher the cost; the fewer candy bars, the lower the cost. We say that the cost (c) varies directly as the number (n) of candy bars. This translates to the algebra equation

c = kn. Where k is the cost of each candy bar.

This is an example of a direct variation.

Indirect

"y" varies indirectly as "x" translates to: $y = \frac{k}{x}$ where k is called the constant of variation.

Suppose we plan to drive to a city 100 miles away. How long will it take us to make the trip? The time will depend on our average rate of speed. The faster we travel, the shorter will be the required time. If we average 25 mph our trip will be 4 hours. If we average 40 mph our trip will be 2 ½ hours. This is an example of inverse variation.

Problems involving variation include a "for instance" situation and then ask for a prediction based on that "for instance".

Examples

y varies directly as x and y= 2.50 when x is 5. What is y when x is 9?

$$y = kx$$

$$2.50 = k(5)$$

In a more algebraic way we would say $k = .50$

Rewriting our variation

$$y = .5x$$

So, when x = 9, y = 4.50

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y varies indirectly as x and y = 80 when x = 3. What is y when x is 12?

y varies indirectly as x and y = 80 when x = 3. Translates to

$$\begin{cases} y = \frac{k}{x} \\ 80 = \frac{k}{3} \\ k = 240 \\ y = \frac{240}{x} \end{cases}$$

When x = 12 ….. $y = \frac{240}{12}$

$$y = 20$$

Direct and inverse variation often occurs together.

"x varies directly as y and inversely as z" would be written as $x = \frac{ky}{z}$. The $x = ky$ is the direct portion. The inverse portion has the same "constant of proportionality" so it is represented as the $y = \frac{k}{z}$.

Combining the two statements we get $x = \frac{k\,y}{z}$.

Thus … if x varies directly as y and inversely as z, and x = 60 when y=15 and z=12 find y when x=24 and z=4.

$$x = \frac{ky}{z}$$
$$60 = \frac{k(15)}{12}$$
$$60 = \frac{k(5)}{4}$$
$$\frac{4}{5} \bullet 60 = k$$
$$48 = k$$
$$x = \frac{48y}{z}$$
$$24 = \frac{48y}{4}$$
$$24 = 12y$$
$$2 = y$$

When we say that x varies jointly as y and z we mean that x varies directly as y and directly as z and write the statement as $x = kyz$. Our method for solving is basically the same as in previous examples.

Variation problems

1. If x varies directly as y and x = 15 when y=20, find x when y = 8.
2. If s varies directly as t and s=22 when t = 55, find t when s = 14.
3. If x varies inversely as y and x=18 when y = 5, find x when y = 15.
4. If m varies inversely as n, and m = 5 when n =3, find n when m=2.
5. If x varies directly as y and inversely as z, and x=8 when y = 12 and x=6, find x when y =16 and z=4.
6. If x varies inversely as y and directly as z, and x = 15 when y =14 and z = 35, find y when x=8 and z=6.
7. If x varies jointly as y and z and x = 24 when y =18 and z=4, find z when x=15 and y=5.
8. If v varies directly as t and inversely as p and v = 15 when t = 273 and p = 20, find p when v =10 and t =546.
9. The volume, v, of a gas varies directly as the temperature, t, and inversely as the pressure, p. If a gas line has a volume of 10 cu ft at a temperature of 270° under a pressure of 15 lb per sq in, find the volume of the gas at a temperature of 810° under a pressure of 90 lb per sq in.
10. If the number of days required to do a job varies inversely as the number of men working and 6 men can do the job in 14 days, how long would it take 8 men to do the job? (hint: $d = \frac{k}{m}$)